

## t-test

- Given by W.S. Gosset in 1908 under the pen name of student's test
- t-test can be applied when:
  1. When a researcher draws a small random sample ( $n < 30$ ) to estimate the population ( $\mu$ );
  2. When the population standard deviation ( $\sigma$ ) is unknown;
  3. The population is normally distributed

Q: Royal tyre has launched a new brand of tyres for tractors & claims that under normal circumstances the average life of tyres is 40000 km. a retailer wants to test this claim & has taken a random sample of 8 tyres. He tests the life of tyres under normal circumstances. The results obtained are:

Tyres	1	2	3	4	5	6	7	8
Km	35 000	38 000	42 000	41 000	39 000	41 500	43 000	38 500

Use  $\alpha = 0.05$  for testing the hypothesis

Step1: Set null & alternative hypothesis

Null hypothesis:  $H_0: \mu = 40\ 000$

Alternative hypothesis:  $H_0: \mu \neq 40\ 000$

Step2: Determine the appropriate statistical test

The sample size is less than 30, so t test will be an appropriate test

Step3: Set the level of significance

The level of significance, i.e.  $\alpha = 0.05$

Step4: Set the decision rule

The t distribution value for a two-tailed test is  $t_{0.025} = 2.365$  for degrees of freedom 7. so if computed t value is outside the  $\pm 2.365$  range, the null hypothesis will be rejected otherwise accepted.

- Step 5: Collect the sample data:

Tyres	1	2	3	4	5	6	7	8
Km	350000	38000	42000	41000	39000	41500	43000	38500

- Step 6: Analyze the data

$\bar{X}=39750$ ;  $\mu=40000$ ;  $s=2618.61$   $n=8$ ;  $df=n-1=7$ ;

Table value of  $t_{0.025,7}=2.365$

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{39750 - 40000}{\frac{2618.61}{\sqrt{8}}} = -0.27$$

- Step 7: Arrive at a statistical conclusion & Business implication

The observed t value is -0.27 which falls within the acceptance region & hence null hypothesis is accepted i.e.  $H_0: \mu = 40\ 000$

# Z-test

- Hypothesis testing for large samples i.e.  $n \geq 30$ ;
- Based on the assumption that the population, from which the sample is drawn, has a normal distribution;
- As a result, the sampling distribution of mean is also normally distributed;

## Application:

1. For testing hypothesis about a single population mean;
2. Hypothesis testing for the difference between two population means;
3. Hypothesis testing for attributes.

## Formula for single population mean (finite population)

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

Where,

$\mu$  = population mean

$\bar{x}$  = sample mean

$\sigma$  = population standard deviation

$n$  = sample size



**Q** A marketing research firm conducted a survey 10 yrs ago & found that an average household income of a particular geographic is Rs 10000. Mr. gupta who recently joined the firm a VP expresses doubts. For verifying the data, firm decides to take a random sample of 200 households that yield a sample mean of Rs 11000. assume that the population S.D is Rs 1200. verify Mr. Gupta's doubts using  $\alpha=0.05$ ?

- Step 1: set null & alternative hypothesis

$H_0: \mu=10000$

$H_1: \mu \neq 10000$

- Step2: Determine the appropriate statistical test

Since sample size  $\geq 30$ , so z-test can be used for hypothesis testing

- Step3: set the level of significance

The level of significance is known ( $\alpha=0.05$ )

- Step4: Set the decision rule

Acceptance region covers 95% of the area & rejection region 5%

Critical area can be calculated from the table ( $\pm 1.96$ )

- Step5: collect the sample data

A sample of 200 respondents yield a sample mean of Rs 11000

- Step6: Analyze the data

$$n=200 \quad \mu=10\,000$$

$$\bar{x}=11000 \quad \sigma=1200$$

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{11000 - 10000}{\frac{1200}{\sqrt{200}}} = 11.79$$

- Step7: Arrive at a statistical conclusion & business implication

Z value is 11.79 which is greater than +1.96, hence null hypothesis is rejected and alternative hypothesis is accepted. Hence Mr. Gupta's doubt about household income was right.